Arrays & Searching Algorithms

Data Structures

- Data structure
  - A particular way of storing and organising data in a computer so that it can be used efficiently
- Types of data structures
  - Based on memory allocation
    - Static (or fixed sized) data structures (Arrays)
    - Dynamic data structures (ArrayList)
  - Based on representation
    - Linear (Arrays/linked lists)
    - Non-linear (Trees/graphs)

Array: motivation

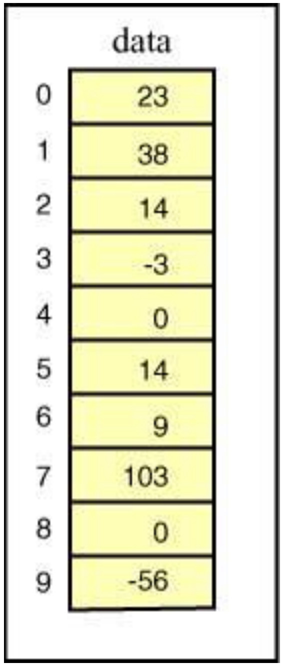
- You want to store 5 numbers in a computer
  - Define 5 variables, e.g. num1, num2, ..., num5
- What, if you want to store 1000 numbers?
  - Defining 1000 variables is a pity!
  - Requires much programming effort
- Any better solution?
  - Yes, some structured data type
    - Array is one of the most common structured data types
    - Saves a lot of programming effort (cf. 1000 variable names)

What is an Array?

- ❑ A collection of data elements in which
  - ▪ all elements are of the same data type, hence homogeneous data
    - o An array of students' marks
    - o An array of students' names
    - o An array of objects (OOP perspective!)
  - ▪ elements (or their references) are stored at contiguous/ consecutive memory locations
- ❑ Array is a static data structure
  - ▪ An array cannot grow or shrink during program execution – its size is fixed

Basic concepts

- Array name (data)
- Index/subscript (0...9)
- The slots are numbered sequentially starting at zero (Java, C++)
- If there are N slots in an array, the index will be 0 through N-1
  - Array length = N = 10
  - Array size = N x Size of an element = 40
- Direct access to an element



Homogeneity

- All elements in the array must have the same data type

Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Index:	0	1	2	3	4
Value:	5.5	10.2	18.5	45.6	60.5

Index:	0	1	2	3	4
Value:	‘A’	10.2	55	‘X’	60.5

Not an array

Contiguous Memory

- Array elements are stored at contiguous memory locations

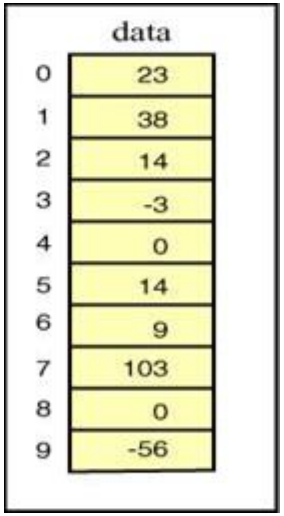
Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

- No empty segment in between values (3 & 5 are empty – not allowed)

Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18		45		60	65	70	80

Using Arrays

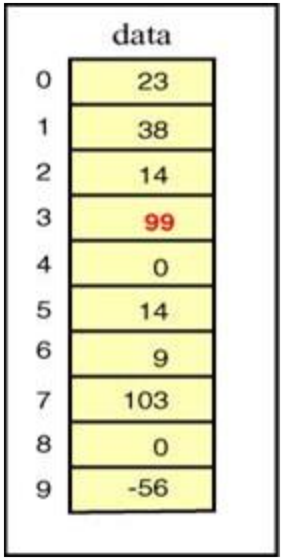
- Array_name[index]
- For example, in Java
  - System.out.println(data[4]) will display 0



data

0	23
1	38
2	14
3	-3
4	0
5	14
6	9
7	103
8	0
9	-56

  - data[3] = 99 will replace -3 with 99



data

0	23
1	38
2	14
3	99
4	0
5	14
6	9
7	103
8	0
9	-56

Some more concepts

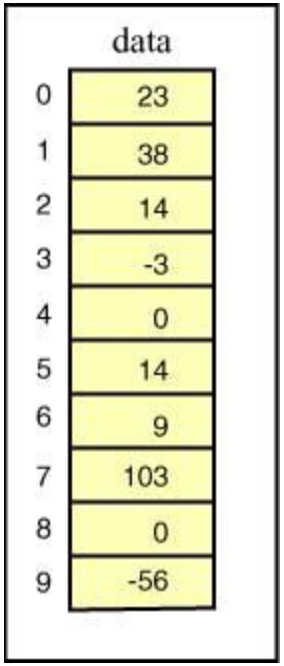
data[-1] always illegal

data[10] illegal (10 > upper bound)

data[1.5] always illegal

data[0] always OK

data[9] OK



data

0	23
1	38
2	14
3	-3
4	0
5	14
6	9
7	103
8	0
9	-56

Q. What will be the output of?

1.data[5] + 10

2.data[3] = data[3] + 10

Array’s Dimensionality

- ❑ One dimensional (just a linear list)
  - e.g.,

5	10	18	30	45	50	60	65	70	80

  - Only one subscript is required to access an individual element

- ❑ Two dimensional (matrix/table)
  - e.g., 2 x 4 matrix (2 rows, 4 columns)

	Col 0	**Col 1**	**Col 2**	**Col 3**
Row 0	20	25	60	40
Row 1	30	15	70	90

Two dimensional Arrays

- ❑ Let, the name of the two dimensional array is M

20	25	60	40
30	15	70	90

- ❑ Two indices/subscripts are required (row, column)
- ❑ First element is at row 0, column 0
  - ▪ $M_{0,0}$ or M(0, 0) or M[0][0] (more common)
- ❑ What is: M[1][2]? M[3][4]?

Array Operations (1)

- Indexing: inspect or update an element using its index. Performance is very fast O(1)

  randomNumber = numbers[5];

  numbers[20000] = 100;

- Insertion : add an element at certain index
  - Start: very slow O(n) because of shift
  - End : very fast O(1) because no need to shift
- Removal : remove an element at certain index
  - Start: very slow O(n) because of shift
  - End : very fast O(1) because no need to shift

Array Operations (2)

- ❑Search : performance depends on algorithm
  - 1) Linear: slow O(n)   2) binary : O(log n)
- ❑Sort : performance depends on algorithm
  - 1) Bubble: slow O(n^2)   2) Selection: slow O(n^2)
  - 3) Insertion: slow O(n^2)   4)Merge : O (n log n)

One Dimensional Arrays in Java

- To declare an array follow the type with (empty) **[]**s

  int[] grade; //or

  int grade[]; //both declare an int array

- In Java arrays are objects so must be created with the **new** keyword
  - To create an array of ten integers:

    int[] grade = new int[10];

    Note that the array size has to be specified, although it can be specified with a variable at run-time

Arrays in Java

- ❑ When the array is created memory is reserved for its contents
- ❑ Initialization lists can be used to specify the initial values of an array, in which case the **new** operator is not used

```
int[] grade = {87, 93, 35}; //array of 3 ints
```

- ❑ To find the length of an array use its **.length** property

```
int numGrades = grade.length; //note: not .length()!!
```

Searching Algorithms (1)

- Search for a target (key) in the search space
- Search space examples are:
  - All students in the class
  - All numbers in a given list
- One of the two possible outcomes
  - Target is found (success)
  - Target is not found (failure)

Searching Algorithms (2)

Index:	0	1	2	3	4
Value:	20	40	10	30	60

Target = 30 (success or failure?)
Target = 45 (success or failure?)
Search strategy?
List Size = N = 5
Min index = 0
Max index = 4 (N - 1)

Sequential Search (1)

- Search in a sequential order
- Termination condition
  - Target is found (success)
  - List of elements is exhausted (failure)

Sequential Search (2)

Index:	0	1	2	3	4
Value:	20	40	10	30	60

Target = 30

Step 1: Compare 30 with value at index 0

Step 2: Compare 30 with value at index 1

Step 3: Compare 30 with value at index 2

Step 4: Compare 30 with value at index 3 (success)

Sequential Search (3)

Index:	0	1	2	3	4
Value:	20	40	10	30	60

Target = 45
Step 1: Compare 45 with value at index 0
Step 2: Compare 45 with value at index 1
Step 3: Compare 45 with value at index 2
Step 4: Compare 45 with value at index 3
Step 5: Compare 45 with value at index 4
Failure

Sequential Search Algorithm (4)

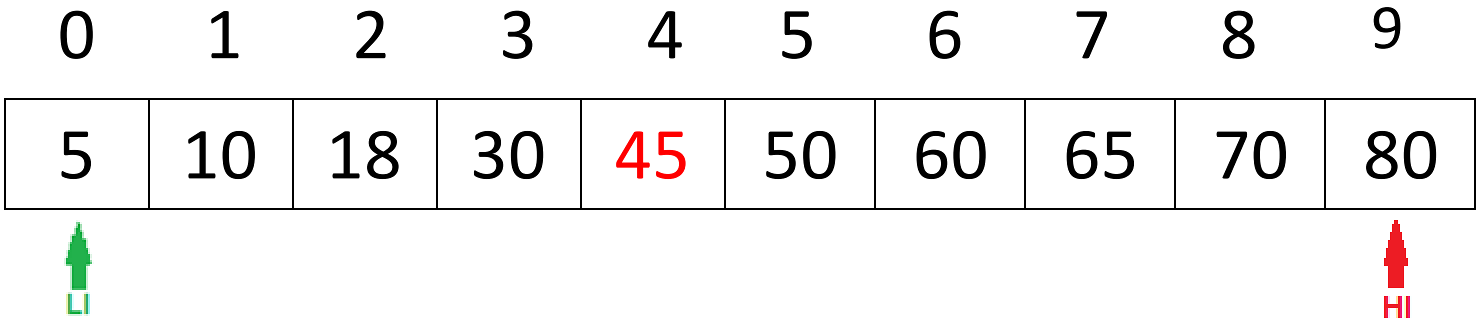
Given: A list of N elements, and the target

```
1. index □ 0
2. Repeat steps 3 to 5
3. Compare target with list[index]
4. if target = list[index] then
           return index // success
     else if index >= N - 1
           return -1 // failure
5. index □ index + 1
```

Binary Search (1)

- Search through a sorted list of items
  - Sorted list is a pre-condition for Binary Search!
- Repeatedly divides the search space (list) into two
- Divide-and-conquer approach

Binary Search: An Example (Key ∈ List) (2)



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Target (Key) = 30

First iteration: whole list (search space), compare with mid value

Low Index (LI) = 0; High Index (HI) = 9

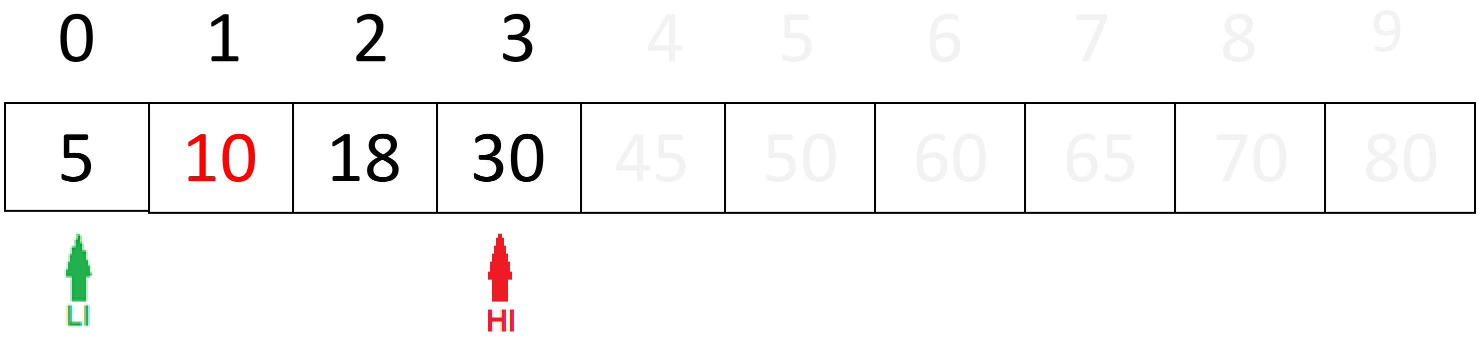
Choose element with index (0 + 9) / 2 = 4

Compare value at index 4 (**45**) with the key (**30**)

30 is less than 45, so the target must be in the lower half of the list

Binary Search: An Example (Key ∈ List) (3)

Second Iteration: Lookup in the reduced search space



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Low Index (LI) = 0; High Index (HI) = (4 - 1) = 3

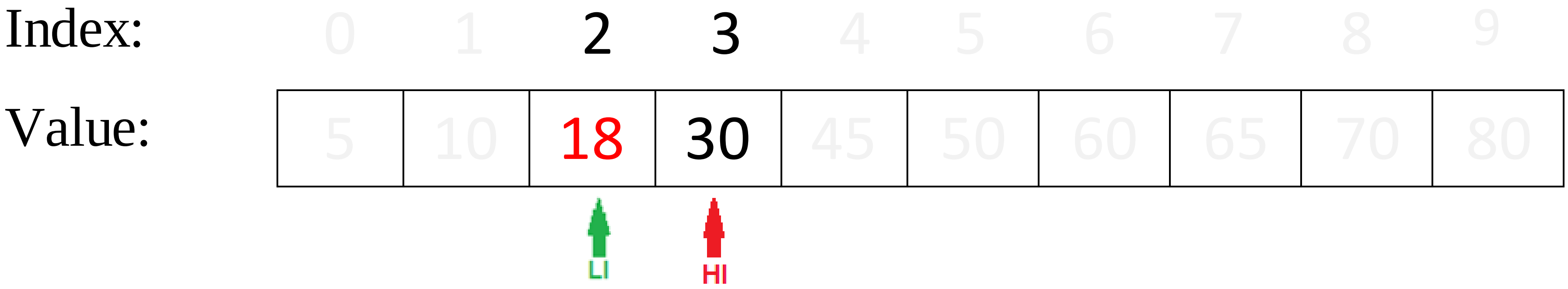
Choose element with index (0 + 3) / 2 = 1

Compare value at index 1 (**10**) with the key (**30**)

30 is greater than 10, so the target must be in the higher half of the (reduced) list

Binary Search: An Example (Key ∈ List) (4)

Third Iteration: Lookup in the further reduced search space



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Low Index (LI) = 1 + 1 = 2; High Index (HI) = 3

Choose element with index (2 + 3) / 2 = 2

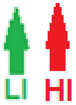
Compare value at index 2 (**18**) with the key (**30**)

30 is greater than 18, so the target must be in the higher half of the (reduced) list

Binary Search: An Example (Key ∈ List) (5)

Fourth Iteration: Lookup in the further reduced search space

Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80



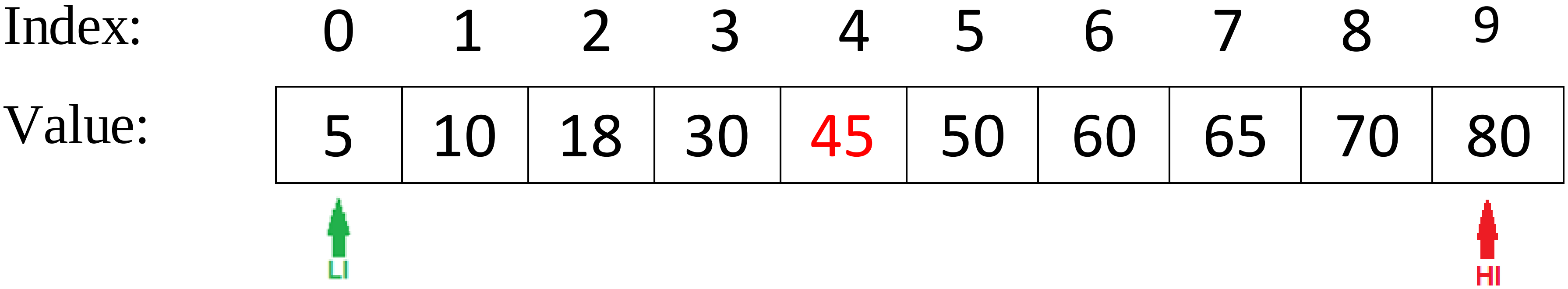
Low Index (LI) = 2 + 1 = 3; High Index (HI) = 3

Choose element with index (3 + 3) / 2 = 3

Compare value at index 3 (**30**) with the key (**30**)

Key is found at index 3

Binary Search: An Example (Key ∉ List) (6)



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Target (Key) = 40

First iteration: Lookup in the whole list (search space)

Low Index (LI) = 0; High Index (HI) = 9

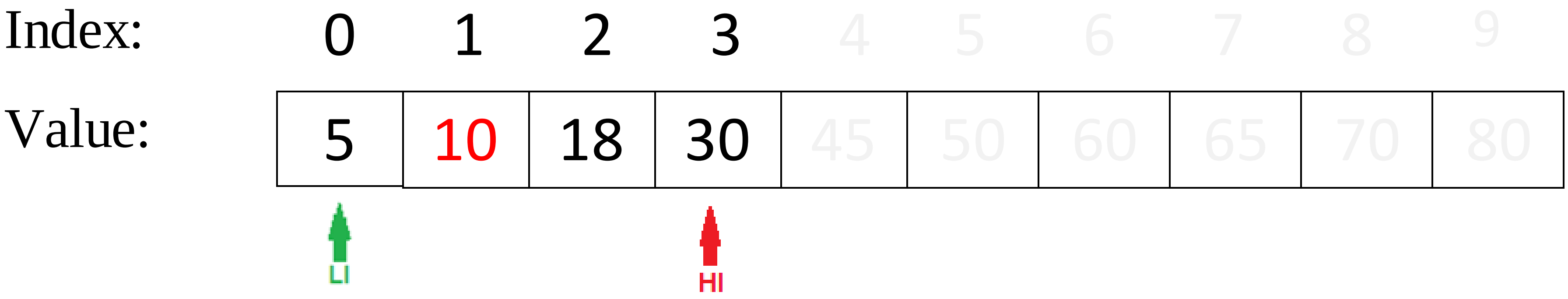
Choose element with index (0 + 9) / 2 = 4

Compare value at index 4 (**45**) with the key (**40**)

40 is less than 45, so the target must be in the lower half of the list

Binary Search: An Example (Key ∉ List) (7)

Second Iteration: Lookup in the reduced search space



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Low Index (LI) = 0; High Index (HI) = (4 - 1) = 3

Choose element with index (0 + 3) / 2 = 1

Compare value at index 1 (**10**) with the key (**40**)

40 is greater than 10, so the target must be in the higher half of the (reduced) list

Binary Search: An Example (Key ∉ List) (8)

Third Iteration: Lookup in the further reduced search space

Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Low Index (LI) = 1 + 1 = 2; High Index (HI) = 3

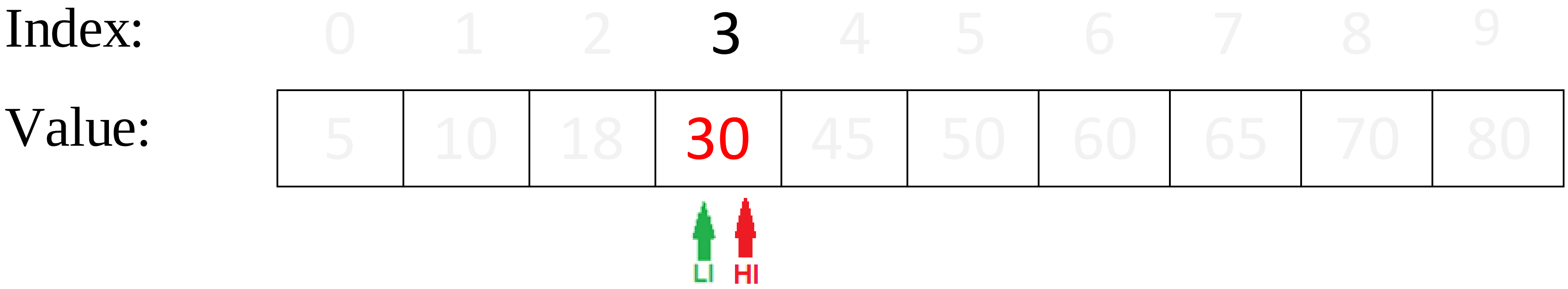
Choose element with index (2 + 3) / 2 = 2

Compare value at index 2 (**18**) with the key (**40**)

40 is greater than 18, so the target must be in the higher half of the (reduced) list

Binary Search: An Example (Key ∉ List) (9)

Fourth Iteration: Lookup in the further reduced search space



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Low Index (LI) = 2 + 1 = 3; High Index (HI) = 3

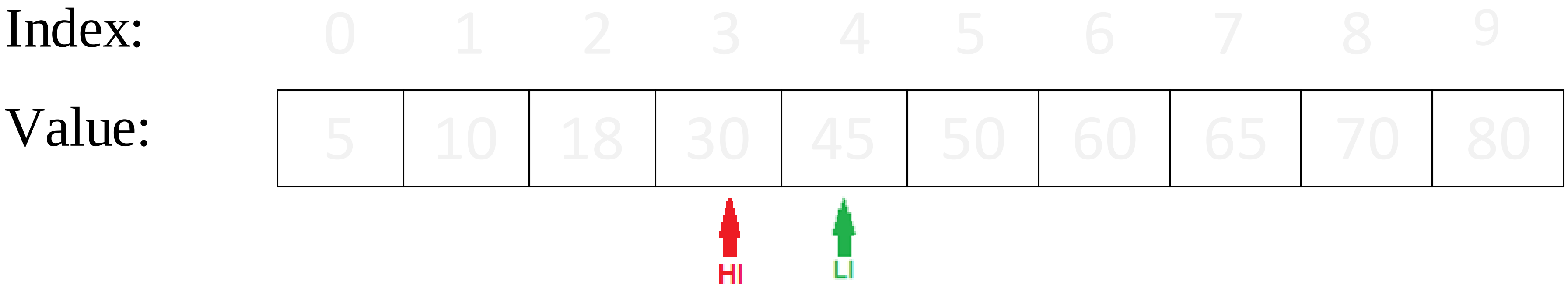
Choose element with index (3 + 3) / 2 = 3

Compare value at index 3 (**30**) with the key (**40**)

40 is greater than 30, so the target must be in the higher half of the (reduced) list

Binary Search: An Example (Key ∉ List) (10)

Fifth Iteration: Lookup in the further reduced search space



Index:	0	1	2	3	4	5	6	7	8	9
Value:	5	10	18	30	45	50	60	65	70	80

Low Index (LI) = 3 + 1 = 4; High Index (HI) = 3

Since LI > HI, Key does not exist in the list

Stop; Key is not found

Binary Search Algorithm: Informal (11)

- Middle □ (LI + HI) / 2
- One of the three possibilities
  - Key is equal to List[Middle]
    - success and stop
  - Key is less than List[Middle]
    - Key should be in the left half of List, or it does not exist
  - Key is greater than List[Middle]
    - Key should be in the right half of List, or it does not exist
- Termination Condition
  - List[Middle] is equal to Key (success) OR LI > HI (Failure)

Binary Search Algorithm (12)

```
❑ Input: Key, List
❑ Initialisation: LI □ 0, HI □ SizeOf(List) – 1
❑ Repeat steps 1 and 2 until LI > HI
    1. Mid □ (LI + HI) / 2
    2. If List[Mid] = Key then
           Return Mid            // success
       Else If Key < List[Mid] then
           HI □ Mid – 1
       Else
           LI □ Mid + 1
❑ Return -1          // failure
```

Search Algorithms:Time Complexity

- Time complexity of Sequential Search algorithm:
  - Best-case : O(1) comparison
    - target is found immediately at the first location
  - Worst-case: O(n) comparisons
    - Target is not found
  - Average-case: O(n) comparisons
    - Target is found somewhere in the middle
- Time complexity of Binary Search algorithm:

  O(log(n)) □ This is worst-case