Algorithms & Data Structures

Dr. Sameer M. Alrehaili
srehaili@taibahu.edu.sa

October 4, 2021

Assignment03 - Solutions

Due Wed 6th Oct 08:00 AM

1 Problem 1

Let A is an array of n elements as depicted in the following.
Write a pseudocode for binary searching over the array, A, which consists of n integer elements?

$$A = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

Algorithm 1: Binary Search

Input: A is a sorted array of n elements $A = (a_1, a_2, \ldots, a_n)$. key, the value of the target element .

Output: An *index* i of the target element such that $k = a_i$, or -1 when it cannot be found.

$l \leftarrow 1$
$r \leftarrow n$
while $(l \leq r)$ **do**
    $mid \leftarrow \lfloor (l + r)/2 \rfloor$
    if $key = A_{mid}$ **then**
        return mid
    else if $key < A_{mid}$ **then**
        $r \leftarrow mid - 1$
    else
        $l \leftarrow mid + 1$
    end if
end while
return -1

$\mathbf{T(n)} = \mathcal{O}(\log n)$ runs in logarithmic time

2 Problem 2

Let A and B be matrices of order $m \times n$, while m is the number of rows and n is the number of columns as they are represented in the following graph.

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ a_{3,1} & a_{3,2} & \cdots & a_{3,n} \\ \vdots & \vdots & \vdots & \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,n} \\ b_{3,1} & b_{3,2} & \cdots & b_{3,n} \\ \vdots & \vdots & \vdots & \\ b_{m,1} & b_{m,2} & \cdots & b_{m,n} \end{pmatrix}$$

Wrtie the pseudocode of two matrices multiplication and estimate its time complexity?
Note: You are not asked to write a Java program. Just write a set of steps to solve the problm of multiplying two matrices.

Algorithm 2: Multiplication

Input: A, B are two matrices.
Output: A new matrix $P = A \times B$.

$m_1 \leftarrow$ the number of rows in A
$m_2 \leftarrow$ the number of rows in B
$n_1 \leftarrow$ the number of columns in A
$n_2 \leftarrow$ the number of columns in B
if $n_1 \neq m_2$ **then**
    print("Matrices cannot be Multiplied ")
    break
end if
$P \leftarrow$ a new matrix of order $m_1 \times n_2$
$i \leftarrow 1$
while $(i \leq m_1)$ **do**
    $j \leftarrow 1$
    while $(j \leq n_2)$ **do**
        $k \leftarrow 1$
        while $(k \leq m_2)$ **do**
            $p_{i,j} \leftarrow p_{i,j} + (a_{i,k} \times b_{k,j})$
            $k \leftarrow k + 1$
        end while
        $j \leftarrow j + 1$
    end while
    $i \leftarrow i + 1$
end while

$\mathbf{T(n)} = \mathcal{O}(n \times n \times n) = \mathcal{O}(n^3)$ runs in polynomial time