

CS112

Recursion (Part 1)

Chapter 18

Lecture 13

الفصل الدراسي الثاني 1443 - Spring 2022

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Introduction

- Suppose you want to find all the files under a directory that contains a particular word. How do you solve this problem?
- There are several ways to solve this problem. Can you give me examples?
- **Recursion**: An intuitive solution is to use recursion by searching the files in the subdirectories recursively.

Case Study – Computing Factorial

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$n! = n * (n-1)!$

- See `ComputeFactorial.java`

Computing Factorial (1/10)

factorial(4)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial (2/10)

$$\text{factorial}(4) = 4 * \text{factorial}(3)$$

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

Computing Factorial (3/10)

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2)\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial (4/10)

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1))\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial (5/10)

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

`factorial(4) = 4 * factorial(3)`

`= 4 * 3 * factorial(2)`

`= 4 * 3 * (2 * factorial(1))`

`= 4 * 3 * (2 * (1 * factorial(0)))`

Computing Factorial (6/10)

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

`factorial(4) = 4 * factorial(3)`

`= 4 * 3 * factorial(2)`

`= 4 * 3 * (2 * factorial(1))`

`= 4 * 3 * (2 * (1 * factorial(0)))`

`= 4 * 3 * (2 * (1 * 1))`

Computing Factorial (7/10)

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1)\end{aligned}$$

Computing Factorial (8/10)

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1) \\ &= 4 * 3 * 2\end{aligned}$$

Computing Factorial (9/10)

`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1) \\ &= 4 * 3 * 2 \\ &= 4 * 6\end{aligned}$$

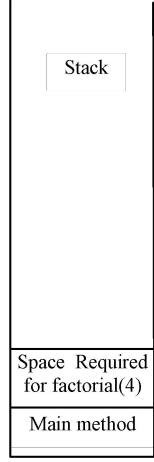
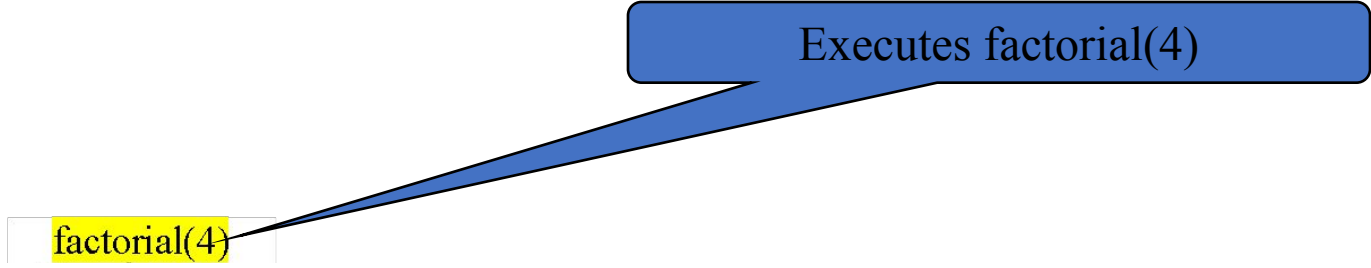
Computing Factorial (10/10)

`factorial(0) = 1;`

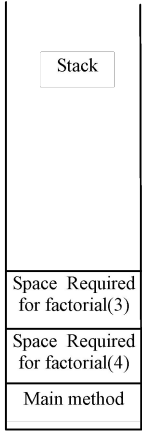
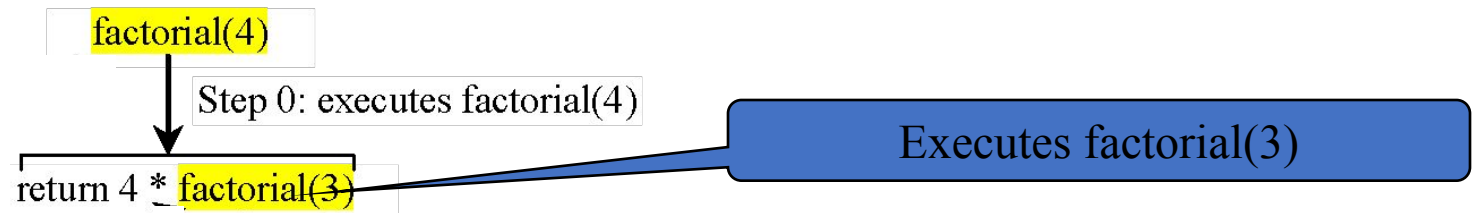
`factorial(n) = n*factorial(n-1);`

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * 3 * \text{factorial}(2) \\ &= 4 * 3 * (2 * \text{factorial}(1)) \\ &= 4 * 3 * (2 * (1 * \text{factorial}(0))) \\ &= 4 * 3 * (2 * (1 * 1)) \\ &= 4 * 3 * (2 * 1) \\ &= 4 * 3 * 2 \\ &= 4 * 6 \\ &= 24\end{aligned}$$

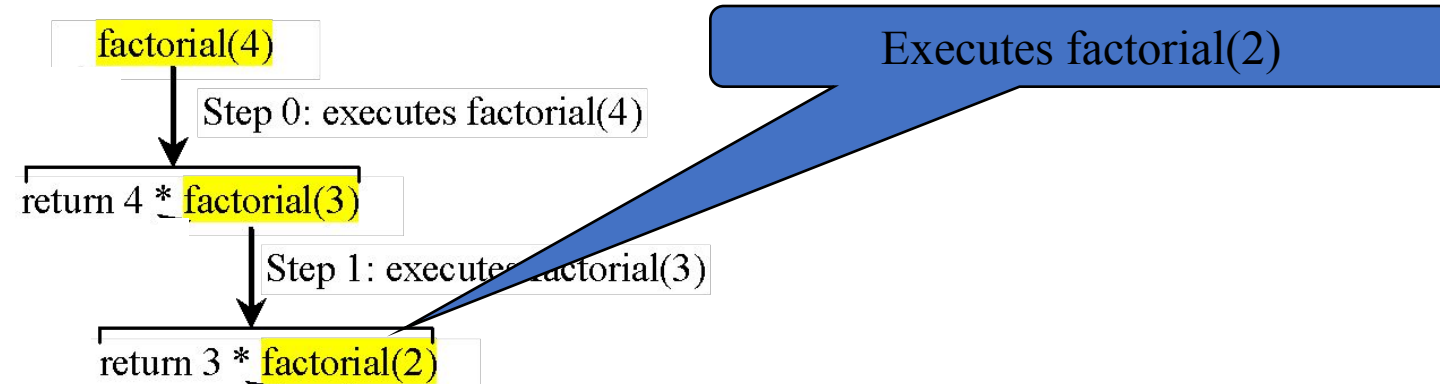
Trace Recursive factorial (1/11)



Trace Recursive factorial (2/11)

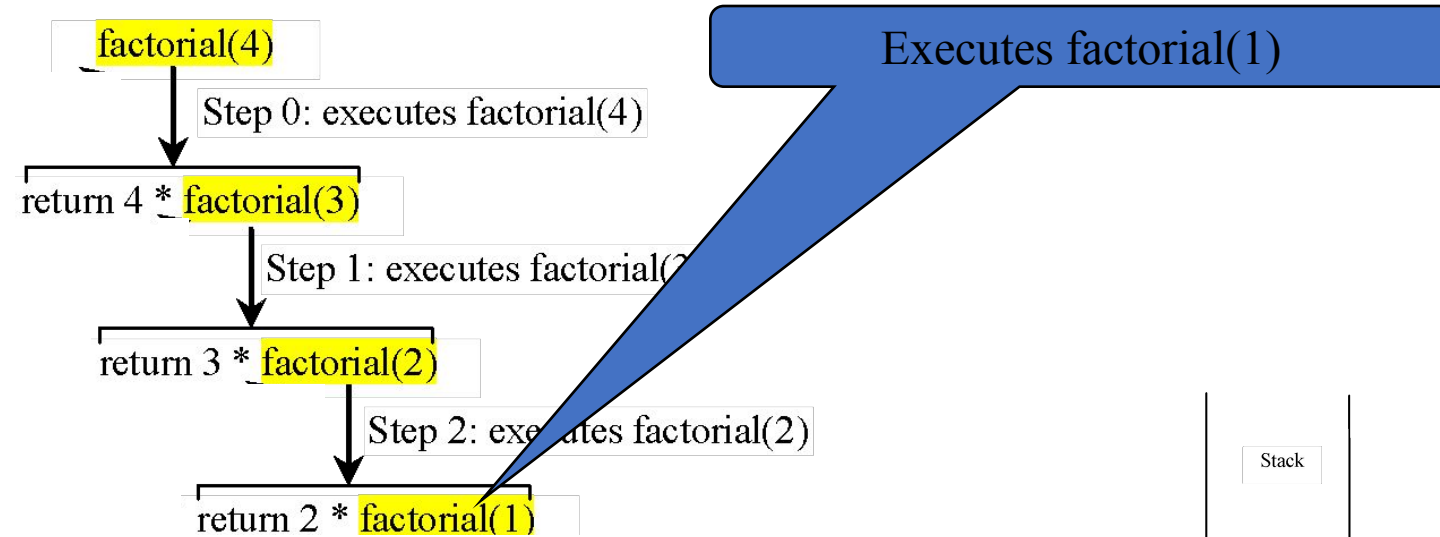


Trace Recursive factorial (3/11)



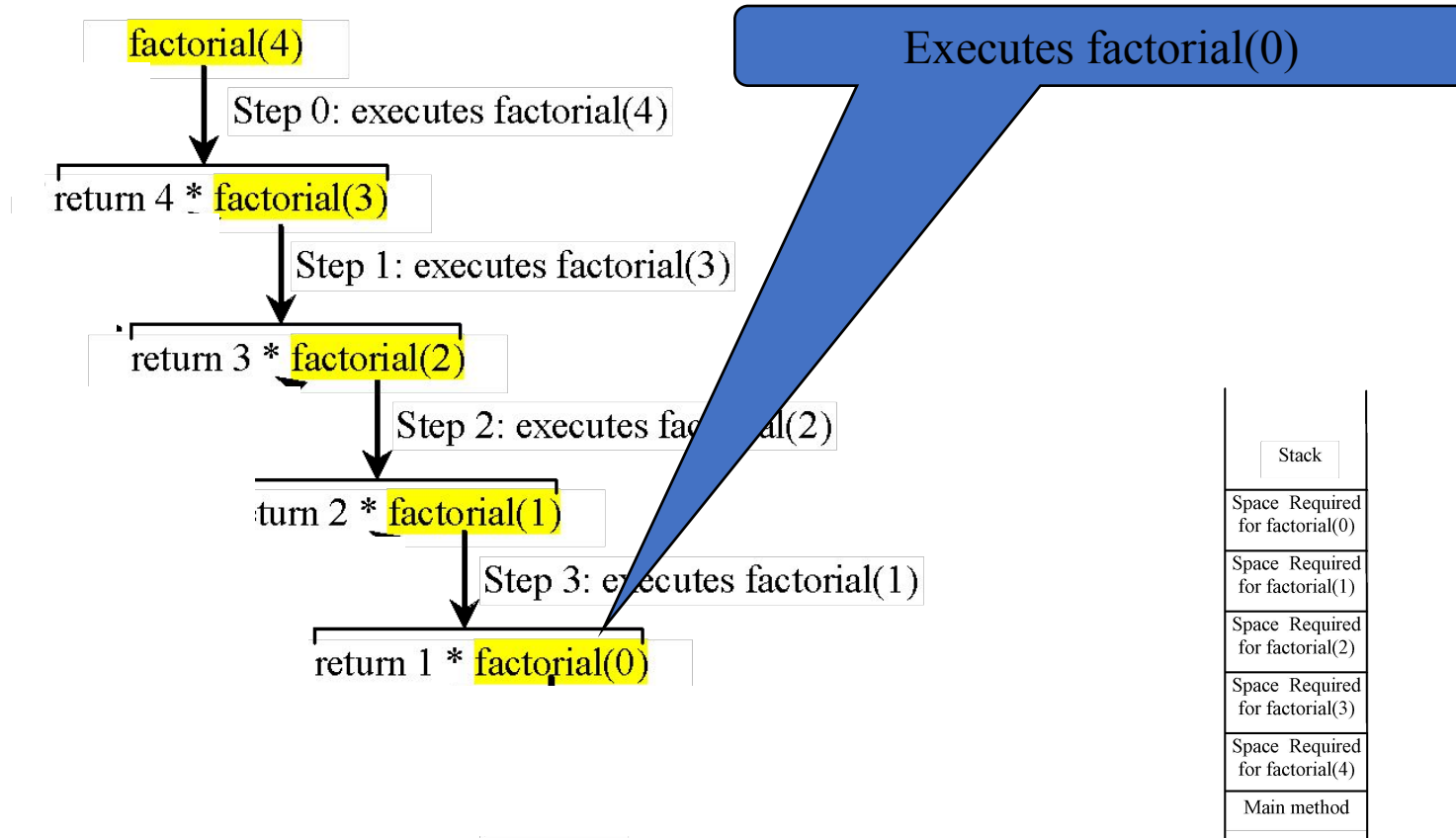
Stack
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Trace Recursive factorial (4/11)



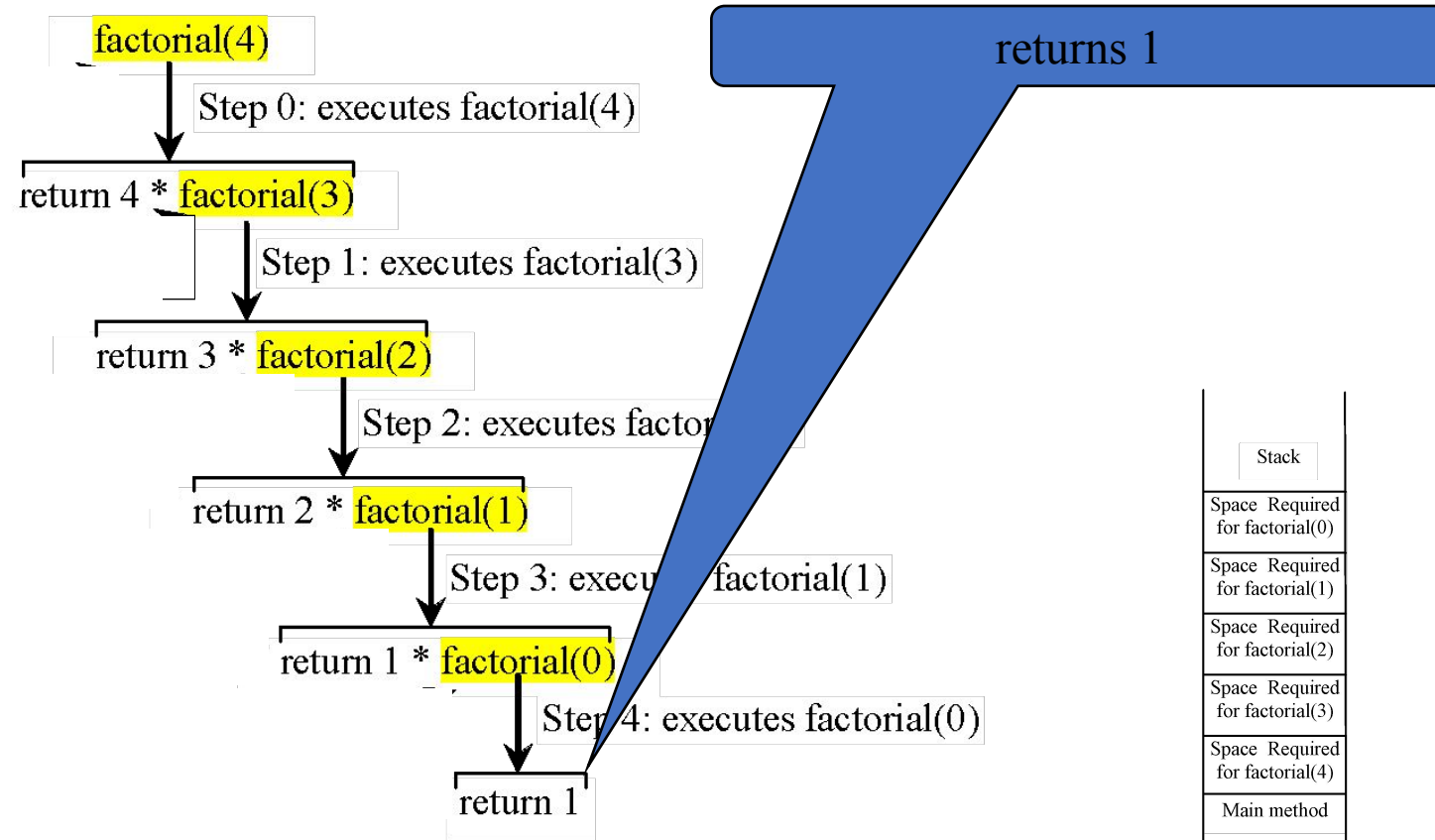
Stack
Space Required for factorial(1)
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Trace Recursive factorial (5/11)

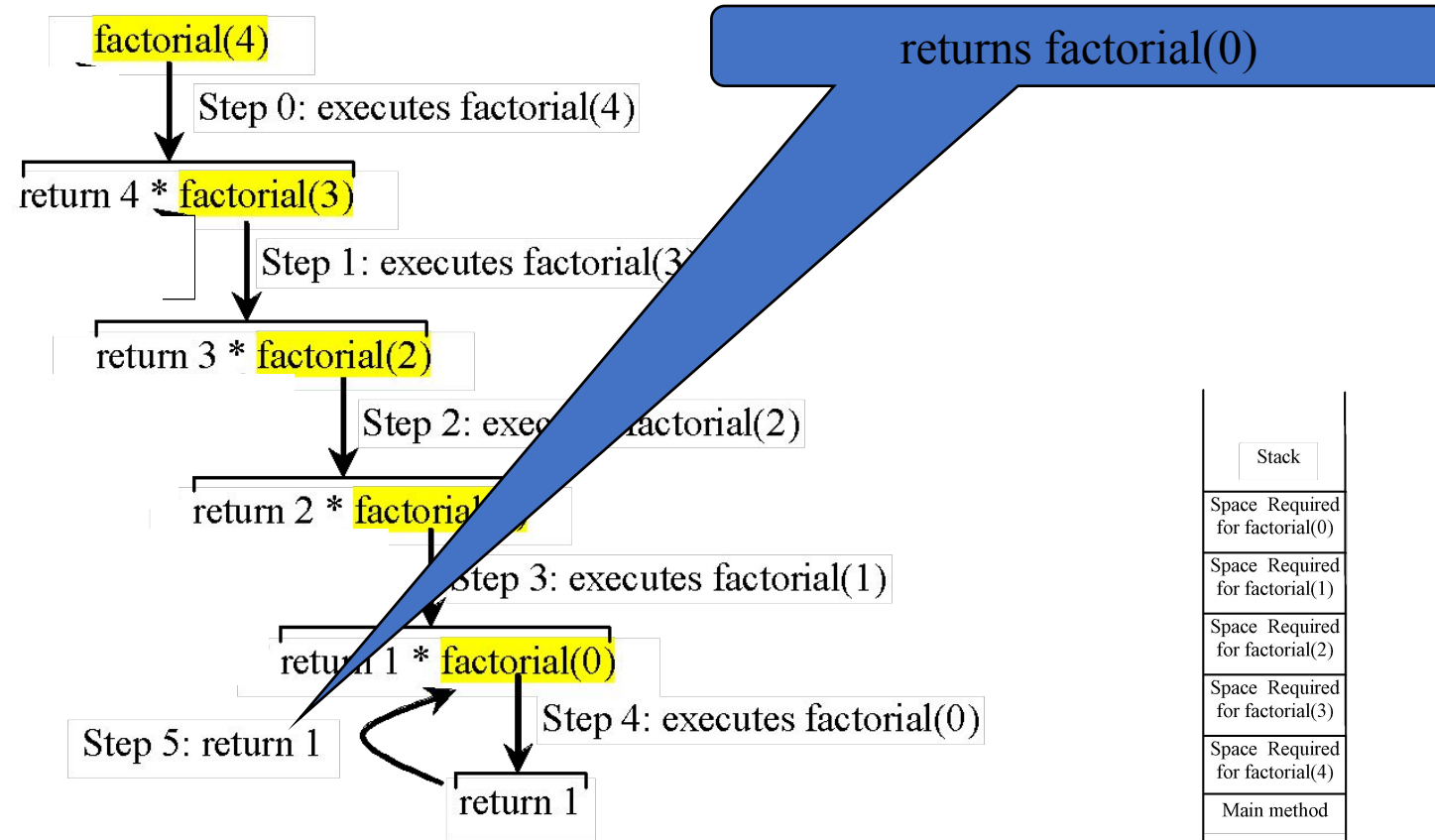


Stack
Space Required for factorial(0)
Space Required for factorial(1)
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

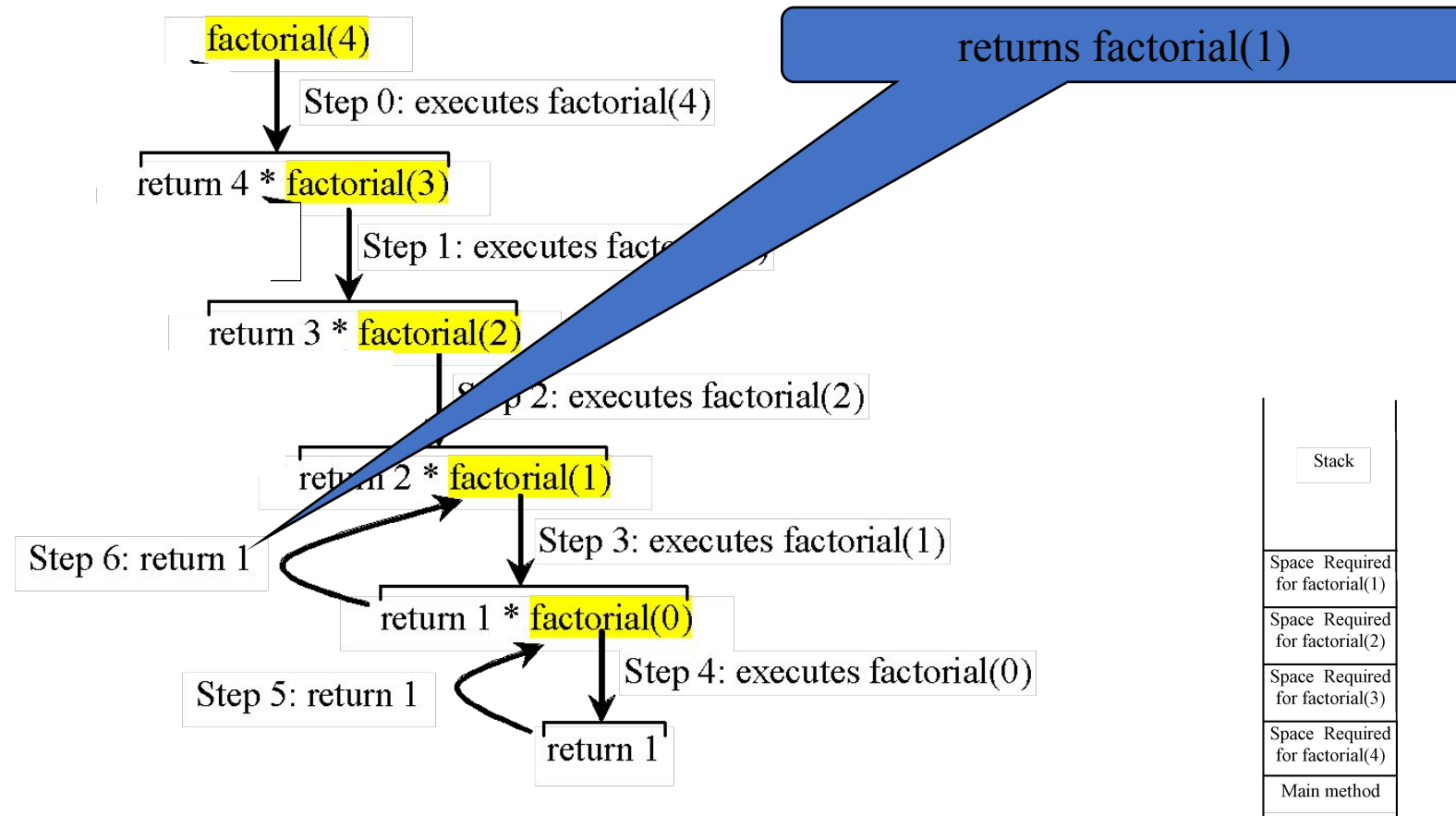
Trace Recursive factorial (6/11)



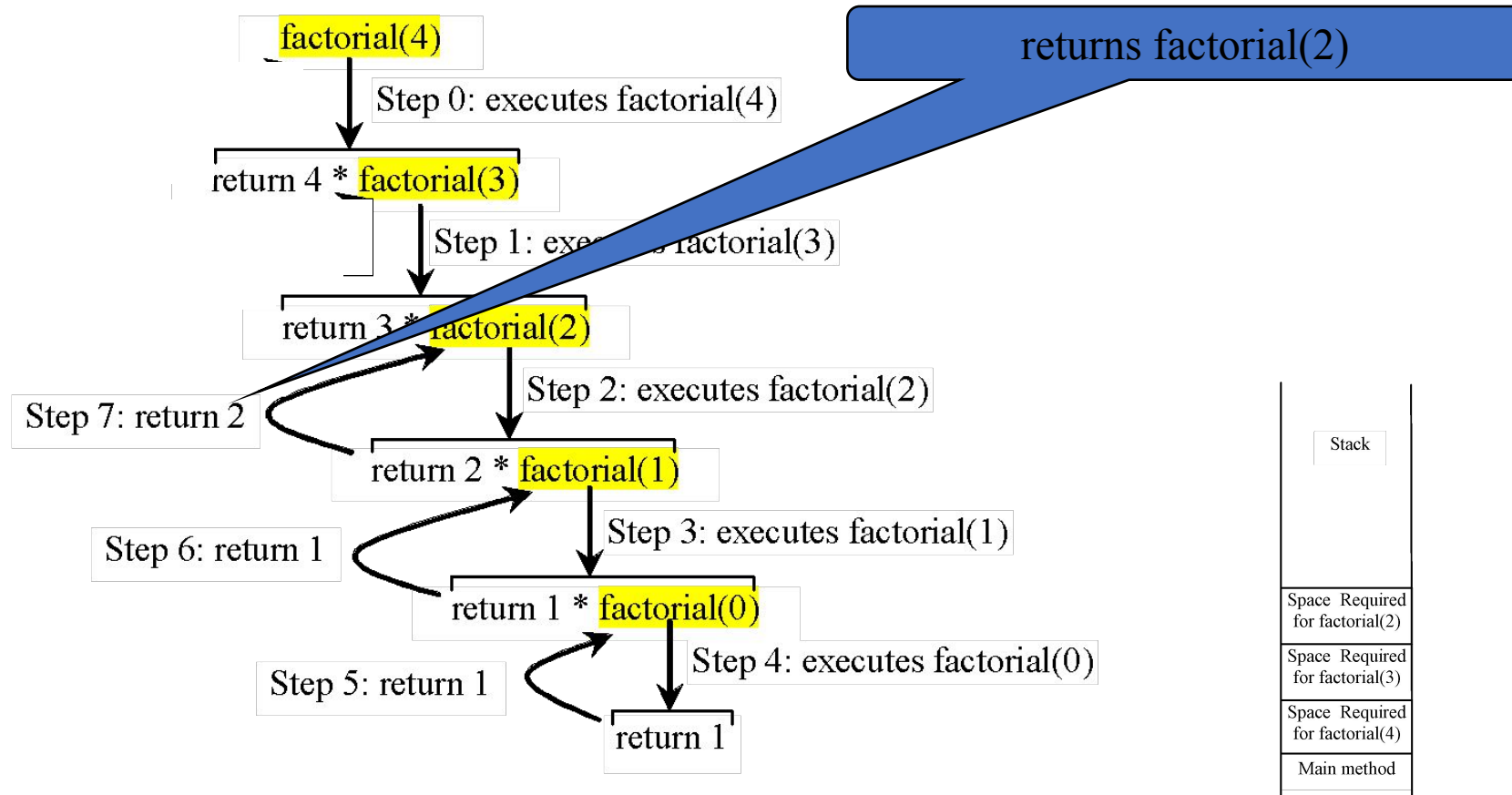
Trace Recursive factorial (7/11)



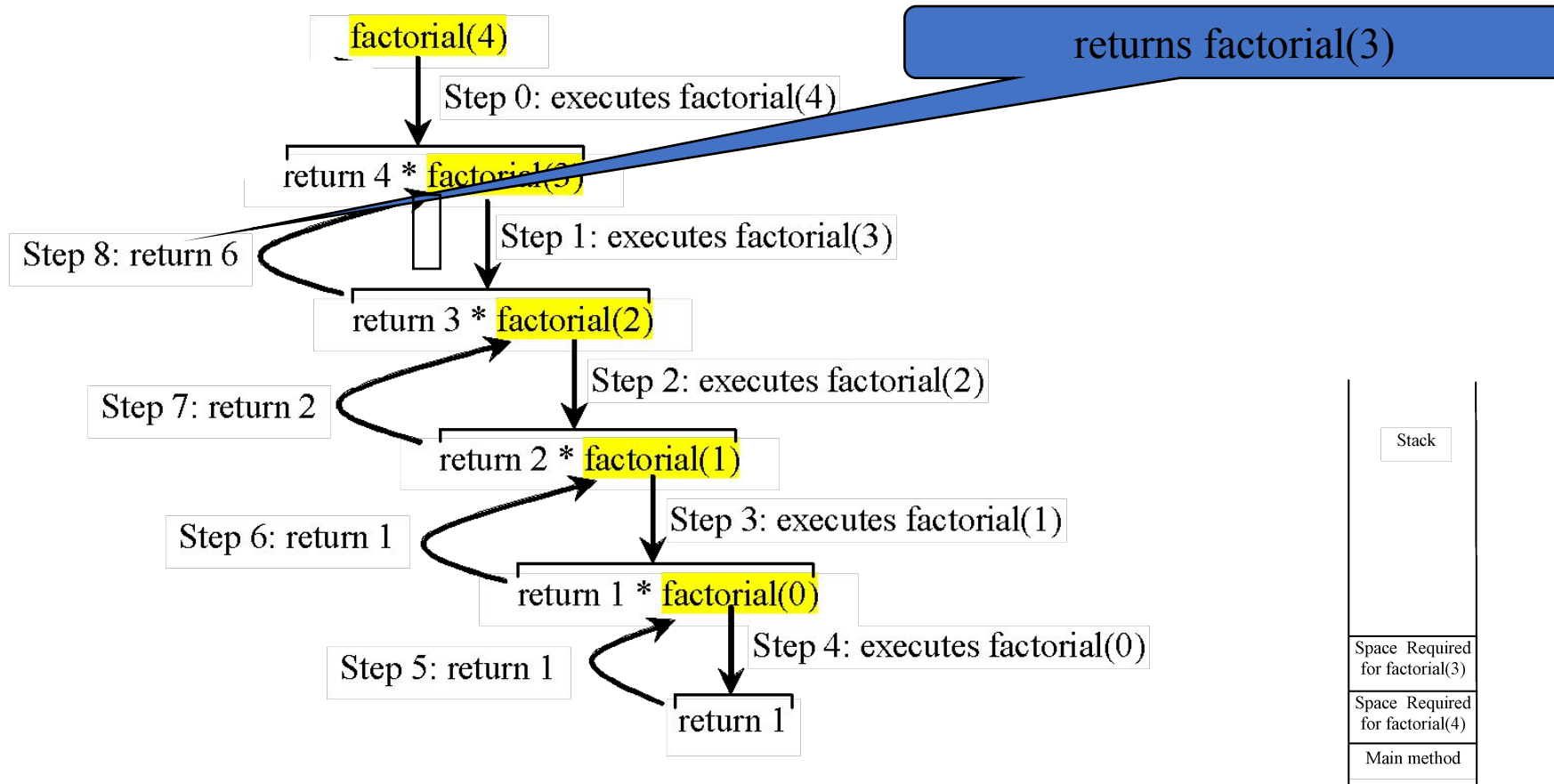
Trace Recursive factorial (8/11)



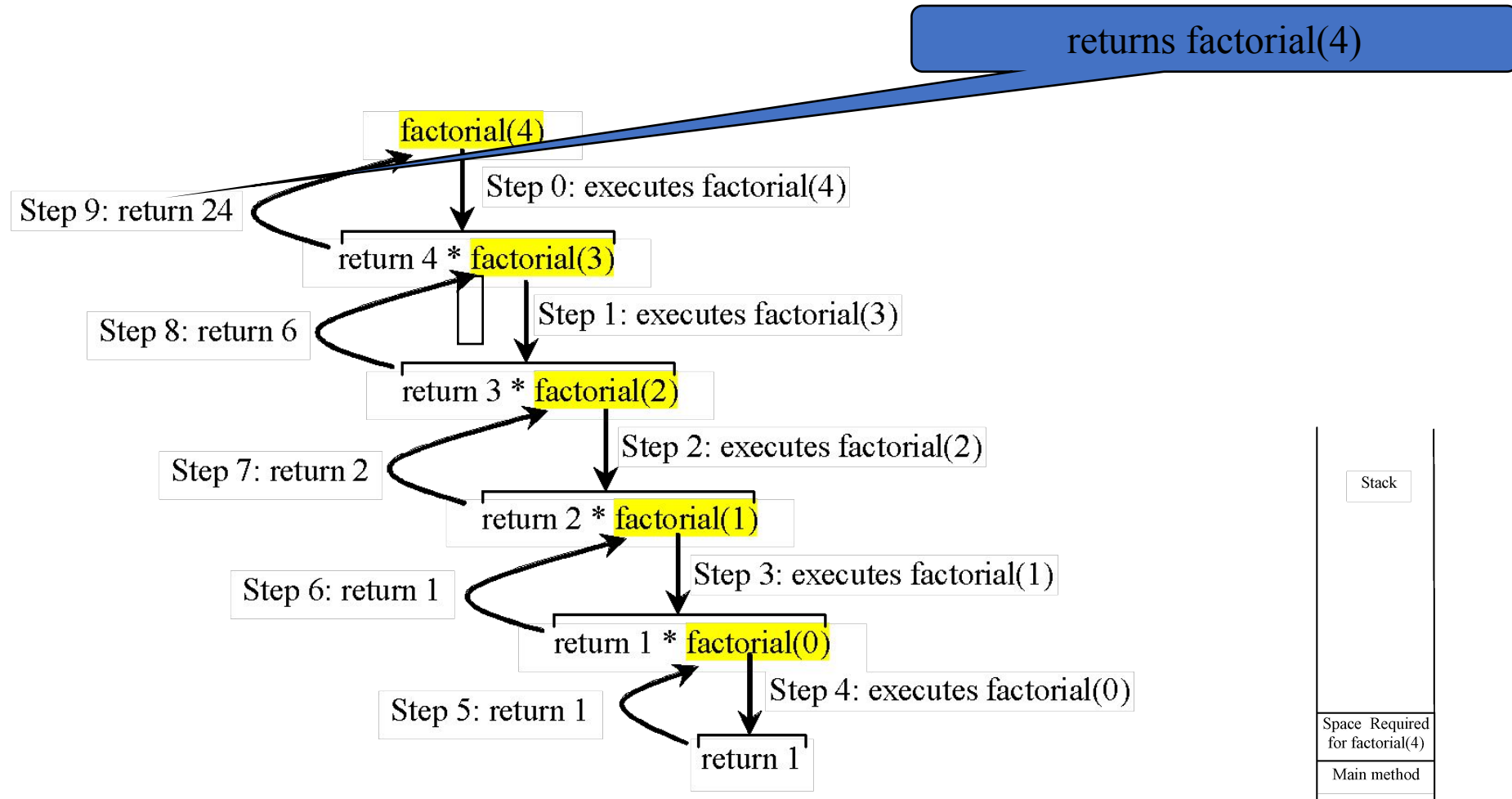
Trace Recursive factorial (9/11)



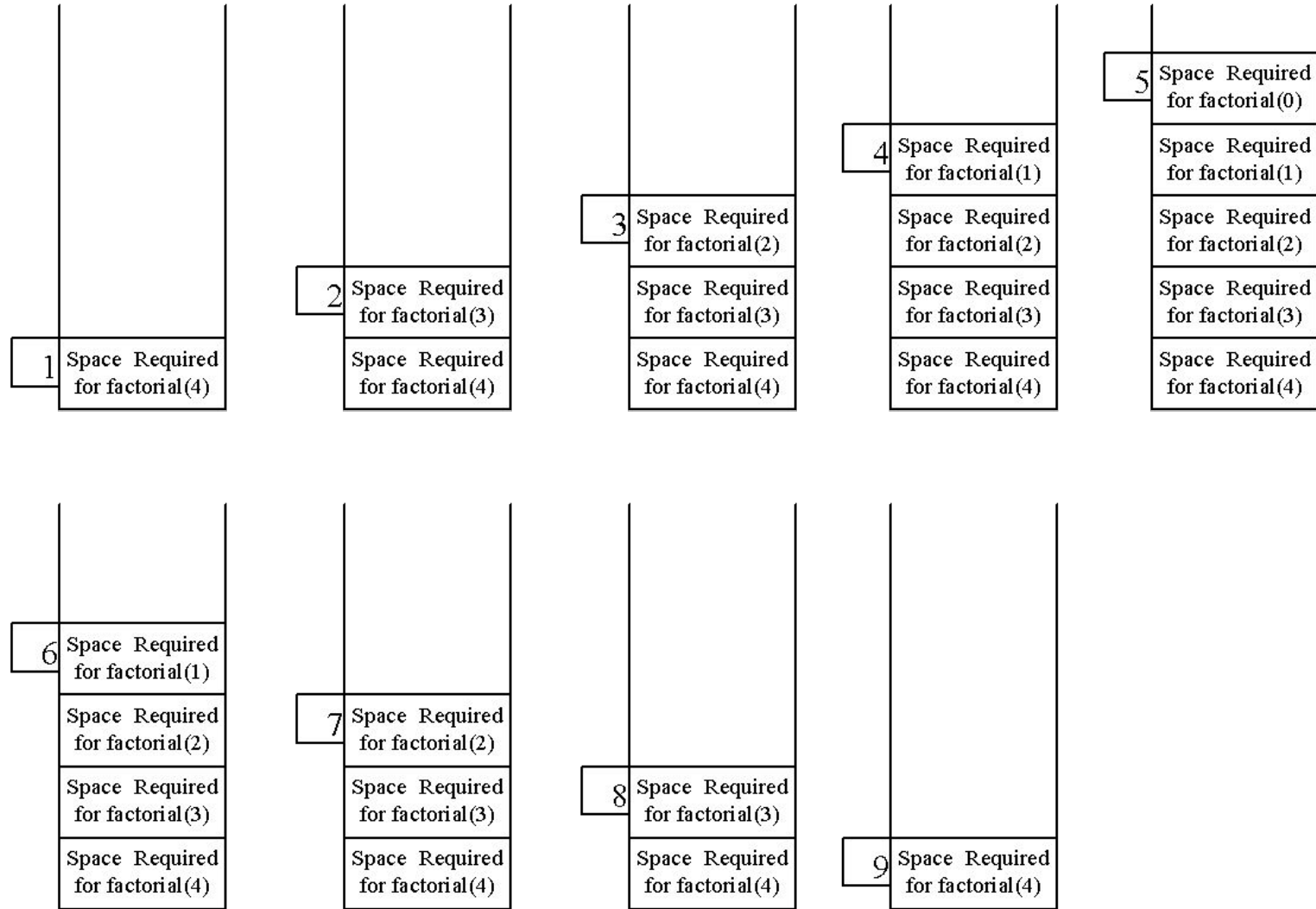
Trace Recursive factorial (10/11)



Trace Recursive factorial (11/11)



factorial(4) Stack Trace



Other Examples

$$f(0) = 0;$$

$$f(n) = n + f(n-1);$$

Case Study - Fibonacci Numbers (1/2)

Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89...

indices: 0 1 2 3 4 5 6 7 8 9 10 11

$\text{fib}(0) = 0;$

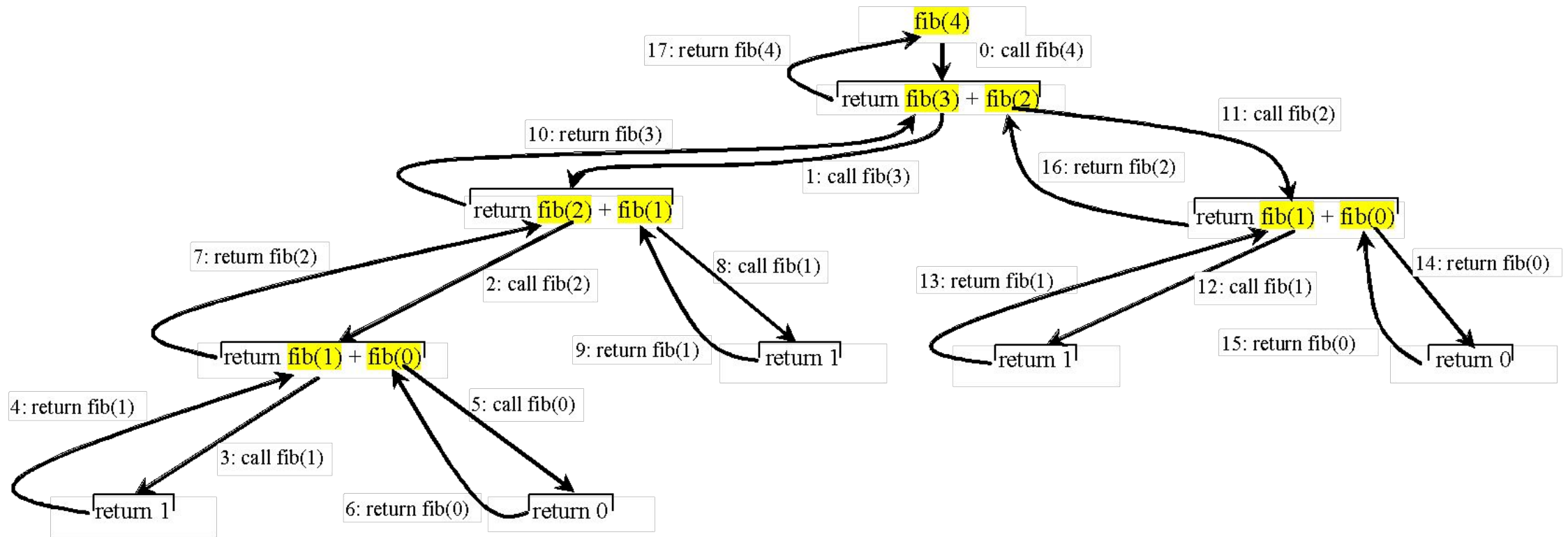
$\text{fib}(1) = 1;$

$\text{fib}(\text{index}) = \text{fib}(\text{index} - 1) + \text{fib}(\text{index} - 2); \text{index} \geq 2$

$\text{fib}(3) = \text{fib}(2) + \text{fib}(1) = (\text{fib}(1) + \text{fib}(0)) + \text{fib}(1) = (1 + 0) + \text{fib}(1) = 1 + \text{fib}(1) = 1 + 1 = 2$

See `ComputeFibonacci.java`

Case Study - Fibonacci Numbers (2/2)



Characteristics of Recursion

- All recursive methods have the following characteristics:
 - One or more base cases (the simplest case) are used to stop recursion.
 - Every recursive call reduces the original problem, bringing it increasingly closer to a base case until it becomes that case.
- In general, to solve a problem using recursion, you break it into subproblems. If a subproblem resembles the original problem, you can apply the same approach to solve the subproblem recursively. This subproblem is almost the same as the original problem in nature with a smaller size.

Problem Solving Using Recursion

Let us consider a simple problem of printing a message for n times. You can break the problem into two subproblems: one is to print the message one time and the other is to print the message for $n-1$ times. The second problem is the same as the original problem with a smaller size. The base case for the problem is $n==0$. You can solve this problem using recursion as follows:

nPrintln("Welcome", 5);

```
public static void nPrintln(String message, int times) {  
    if (times >= 1) {  
        System.out.println(message);  
        nPrintln(message, times - 1);  
    } // The base case is times == 0  
}
```